Comparison of Analytical Models for the Analysis and Design of Series Double Excitation Synchronous Machines

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Abstract — This paper presents some methods to overcome numerical problems met when analytical models based on the formal resolution of Maxwell's equations are used for the analysis and design of series double excitation machines. Three analytical models are compared for predicting cogging torque and eddy current loss in permanent magnets and armature windings.

I. Introduction

This paper compares analytical models for predicting cogging torque and eddy current losses in series double excitation synchronous machines (Fig. 1). Two analytical models are compared for predicting cogging torque and open circuit eddy current losses in permanent magnets and in armature windings, and three models are compared for the prediction of eddy current loss in permanent magnets due to armature reaction field.

The first model is the more complicated one; it is based on the resolution of Maxwell's equations in a machine geometry which can be considered as the most realistic one (Fig. 1) [1]. In the second model the rotor saliency is neglected (no rotoric slots, $R_0 = R_1$) [2] [3] [4]. The third and last model is even simpler since both rotor and stator saliencies are neglected (no stator or rotor slots) [5] [6]. Finite element analyses will provide the reference for comparison (Fig. 2).

Through out this study, it is shown that for the prediction of some electromagnetic performances, simpler analytical models can be used to overcome numerical problems accruing when more complicated analytical models are used.

II. Magnetic Field Analytical Solution

Figure 1 shows the different regions (stator slots (I), airgap (II), permanent magnets (III), rotor slots (IV)) where the exact analytical solution is established thanks to separation of variables method. The model is formulated in two-dimensional polar coordinates. The analytical solution for the magnetic field distribution is established based on following assumptions: 1) the stator and rotor cores are assumed to be infinitely permeable; 2) eddy current effects are neglected (no eddy current loss in the magnets or armature windings); 3) the permeability of permanent magnets is assumed to be equal to that of air; and finally, 4) the axial length of the machines is infinite so that the end effects are neglected.

The partial differential equation for quasi-stationary magnetic fields in a continuous and isotropic region can be expressed in terms of the magnetic vector potential **A**, subject to the Coulomb gauge, $\nabla \times \mathbf{A} = 0$, by

$$\begin{cases} \nabla^2 \mathbf{A} = -\mu_0 \mathbf{J}, & \text{in regions I} \\ \nabla^2 \mathbf{A} = 0, & \text{in region II} \\ \nabla^2 \mathbf{A} = -\mu_0 \nabla \times \mathbf{M}, & \text{in region III} \\ \nabla^2 \mathbf{A} = -\mu_0 \mathbf{J}_f, & \text{in regions IV} \end{cases} \quad (1)$$

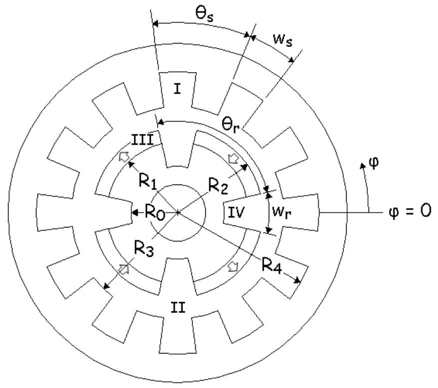


Fig. 1. Idealized series double excitation machine model (polar coordinates).

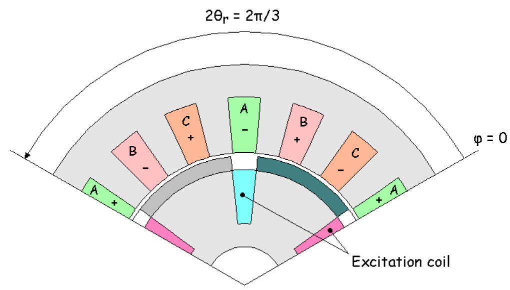


Fig. 2. Finite element model.

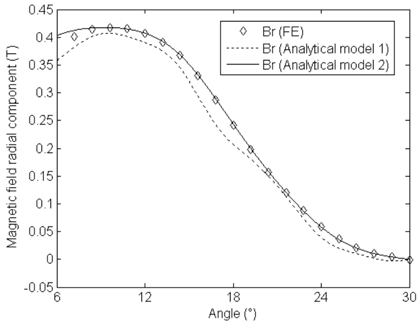
A (the magnetic vector potential) only has A_z component which is independent of z (infinitely long machine in axial direction). **J** is the armature current density vector, **M** is the magnetization and $\mathbf{J}_f$ is the DC field current density.

Combining equations (1) with boundary conditions, and using separation of variables method, help establish a set of linear equations (N_H x N_H) (where N_H is the number of considered harmonics), where coefficients of magnetic vector potential solution in region III are the unknown. Solving these linear equations and using interface conditions give coefficients of magnetic vector potential in other regions. Obtained linear equations are solved using Gaussian elimination method. More details about the developed model can be found in [1].

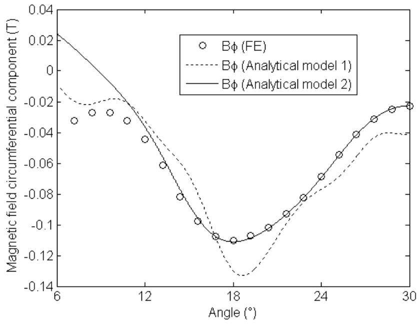
III. Numerical Limitations and Solutions

In order to illustrate numerical limitations inherent to developed model (first model), figure 3 shows a comparison of armature reaction magnetic field components in a permanent magnet obtained by the first analytical model, the second analytical model and finite element computation respectively. The same number of harmonic is considered for the first and second analytical models.

As can be seen, the second analytical model gives more accurate results than the first model in a large part of the permanent magnet region even if rotor saliency is neglected in the second model.



(a) Radial component



(b) Circumferential component

Fig. 3. Comparison of magnetic field space distributions obtained by respectively finite element, first and second analytical models in a magnet.

The difference between the finite element results and the first analytical model are due to numerical limitations inherent to the analytical model [7]. The combination of magnetic field components solutions with boundary conditions results in a set of linear equations which may be ill-conditioned, hence, the solution may become inaccurate.

IV. Calculation of Cogging Torque

Figure 4 shows comparison of cogging torque waveforms, when excitation current is null ($J_f = 0$ A/mm^2), obtained by first analytical model, second analytical model and finite element analysis, respectively. The same number of harmonic is considered for the first and second analytical models. As can be seen, the second analytical model is again giving relatively more accurate results than the first model.

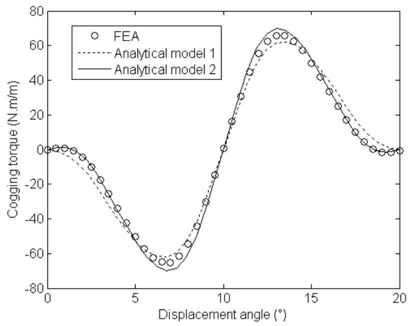


Fig. 4. Cogging torque waveforms comparison.

V. Calculation of Eddy Current Losses

Figure 5 compares open circuit eddy current loss when excitation current is null ($J_f = 0$ A/mm^2) for different values of electrical frequency obtained respectively by finite element analysis and the first and second analytical models respectively.

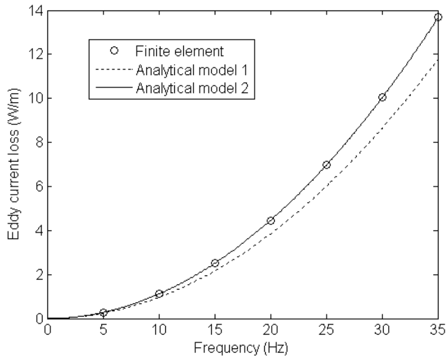


Fig. 8. Comparison of eddy current loss in a magnet obtained by the two analytical models and finite element analysis ($J_f = 0$ A/mm^2).

Once again, the second analytical model is giving more accurate results than the first one as compared to finite element computations. It should be noticed that the same number of harmonic is considered for both analytical models.

VI. Conclusion

This paper compares different analytical models for predicting some electromagnetic performance in series double excitation machines. The goal of this study is to address two problems related to the use of analytical models for design and analysis purposes. The first is to find methods helping to overcome numerical problems met when analytical models based on the formal resolution of Maxwell's equations are used for the analysis and design of series double excitation machines. The second goal is to find methods allowing speeding up of the pre-design and analysis of series double excitation machines using analytical models.

The full paper will contain more details about the proposed approach and give more results to support this approach.

VII. References

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